ME 4201: Mechanical Engineering Design Laboratory

Formal Group Lab Report 6
Second Order System Experiment

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Abstract (Mallory)

The objective of this experiment is to use the response behavior of a system to determine physical properties of the components. The system consists of springs, dampers, and masses that are able to be connected in various cases and scenarios. Together these components create a second-order system as a result of a step input, the carriage being displaced 2.5 cm. As predicted, each case was underdamped, allowing oscillations in displacement over time until the system reached steady state. By studying each case, the natural frequency increased as the spring stiffness increased and decreased as the mass increased. The smaller the mass and the lower the stiffness of the spring, the higher the damping ratio of the system. The mass of carriage 1 and 2 was calculated to be 0.567 kg and 0.548 kg. The heavy spring had a spring constant of 606.906 N/m, the medium spring 314.659 N/m, and the light spring 165.911 N/m. The damping coefficient of case 1, case 2, and case with the dashpot was 2.157 Ns/m, 1.734 Ns/m, and 10.000 Ns/m. The cases with the highest damping ratio were found to return to steady state the fastest and a 477% increase in the damping coefficient was found by attaching a dashpot to the carriage.

Table of Contents

List of Tables

List of Figures

Introduction (Mallory)

The objective of this experiment is to model the behavior of a system to determine physical properties of the components [1]. The system consists of springs, dampers, and masses that can be connected in seven cases. Together these components create a second-order system with a step input. A second order system is any system whose behavior is modeled in terms of first and second derivatives [1]. Second order systems are commonly studied in engineering. An example of a second-order system is automobile suspension [2]. This design utilizes the system to dampen bumps and potholes in the road to provide a smooth ride for passengers. Simple springs are used to model many different complicated systems because they will react in a similar way for example atoms in crystals, pendula, and balls rolling in a bowl [3]. This behavior is commonly referred to as simple harmonic motion [3].

The experiment uses a rectilinear plant with carriages, dampers, and springs to track the displacement of the carriages plotted against time. Using these tables, the damping ratio, natural frequency, and damping frequency can be calculated [1]. Physical properties could be determined for the system such as the mass of each carriage, spring constant of different springs used, and damping coefficient of carriages. This experiment allows students to understand behavioral responses in second order systems and determine the masses, spring coefficient, and damping coefficient of the system.

Background and Theory (Mallory)

The behavior of a second-order system consists of second and first order derivatives. This system is the lowest order system that can produce an oscillatory response with a step input [4]. The general form of a second-order system of equations can be seen in Equation 1.

$$\frac{1}{{\omega_n}^2}\ddot{x} + \frac{2\zeta}{\omega_n}\dot{x} + x = \frac{1}{k}F(t) \qquad \textit{Equation } 1 \text{ [1]}$$

The example used in this experiment is a mass-spring-damper system [1]. This common system is modeled with Equation 2 below were m is the mass, c is the damping coefficient, k is the spring constant, x is the linear displacement and its higher powers, and F is the applied force [1]. The natural frequency and damping force, discussed more in this report, can be found for a mass-spring-damper system with Equations 3 and 4 below where ω_n is the natural frequency and ζ is the damping ratio [1].

$$F(t) = m\ddot{x} + c\dot{x} + kx \qquad \textit{Equation } 2 \text{ [1]}$$

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \textit{Equation } 3 \text{ [1]}$$

$$\zeta = \frac{c}{2\sqrt{mk}} \qquad \textit{Equation } 4 \text{ [1]}$$

The spring in the second order system in this experiment produces a force that increases linearly with displacement; however, nonlinear springs do exist. Nonlinear springs are seen in materials during plastic deformation or large deflecting beams [1]. Springs in a neutral position do not produce any force along their length until they compressed or stretched. The work to expand or compress the spring is transferred into potential energy during this process [3]. The repeated compressing and stretching of springs are referred to as an oscillation, and the oscillations per unit time is the frequency of the response.

Damping is a force that resists motion. Damping can exist as a dashpot where a fluid is forced through a small opening or friction on a sliding box [1]. Damping can be harmful to a system depending on the physical system for example when seeking to minimize vibration or drag against large trucks. Damping can be linear or nonlinear. Nonlinear damping occurs when aerodynamic drag has large Reynolds numbers and linear damping occurs at smaller Reynolds numbers [1]. Sliding friction is a concept most individuals have experienced and can be quantified in Equation 5 where F_N is the normal force, μ is the coefficient of friction.

$$F_{sliding\ friction} = -\mu F_N sgn(\dot{x}) \qquad Equation\ 5\ [1]$$

The figure below shows a mass-spring-damper free body diagram. The masses represent carriages in this experiment subject to damping due to friction and in some cases a dashpot.

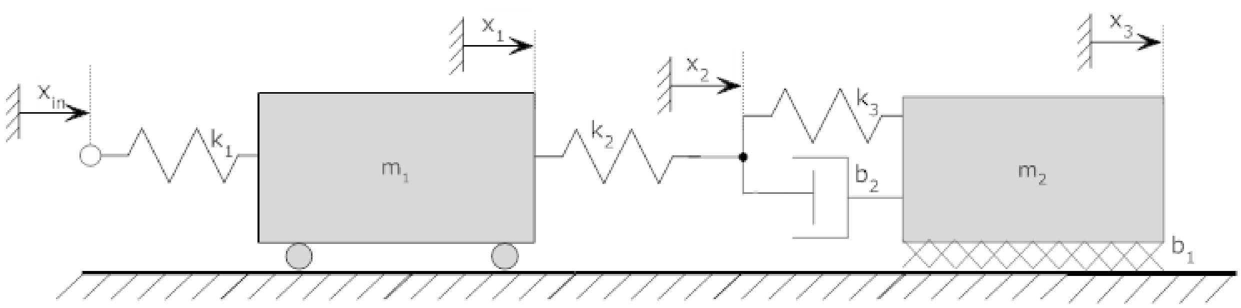

Figure 1: Free Body Diagram of Mass-Spring-Damper System [5]

In a spring-mass system with no loss, a system would oscillate at its natural frequency indefinitely. However, real life has friction loss, air resistance, and many mechanisms of removing energy from the system. The damping ratio is a dimensionless value that quantifies the rate of amplitude dampening in a system independent of time [4]. This value, “compares the exponential decay frequency of the envelope to the natural frequency” [4].

The damping ratio is categorized into four groups. The damping ratios with a value equal to zero is an idealized system without any losses and called an undamped system. This system will oscillate forever without any dampening. The second category encompasses damping ratios from zero to one called underdamped responses. These responses will oscillate before settling in a steady state position. Critically damped responses are those where the damping ratio is one. These systems will not oscillate and has the fastest decay of any case [6]. Over damped systems have a damping ratio greater than one. A real-life example of overdamping is a door damper that is commonly seen at the top of the door and utilized when closing. When the door damper is overdamped, it will slowly settle into a closed position without oscillating [6]. The image below displays the difference in behavior between undamped, underdamped, critically damped, and overdamped systems.

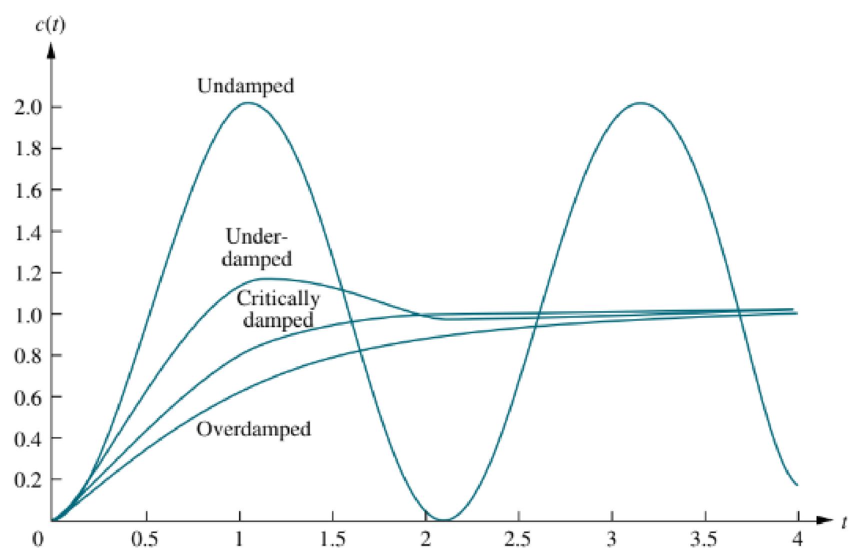

Figure 2: Step Responses for Second-Order System Damping Cases [4]

Underdamped second-order systems are common for modeling physical phenomena. In this experiment, it is predicted that the response will be underdamped systems of varying damping ratios. The image below indicates how the underdamped response changes with damping ratio approaching one.

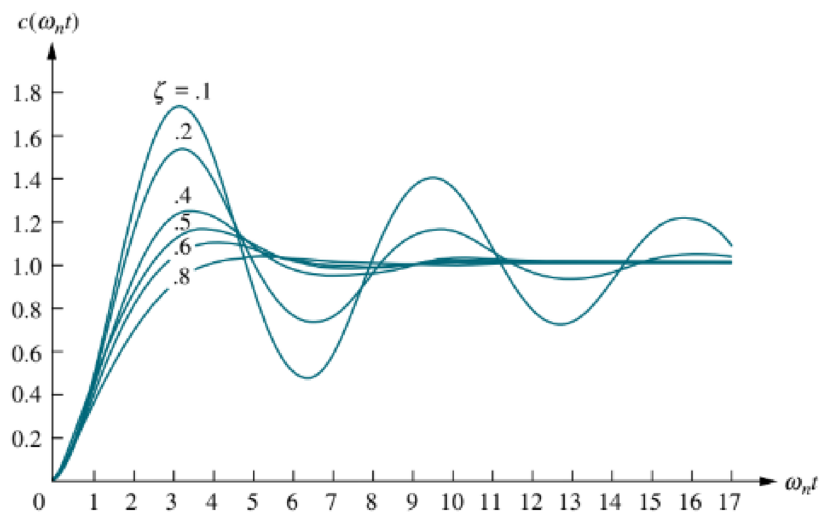

Figure 3: Second-Order Underdamped Responses for Damping Ratio Values [4]

As shown in Figure 2, an underdamped system will exhibit decaying oscillations in response to a step input $F(t)$ where the frequency is called damped or ringing frequency, ω_d [1]. The equation below shows how this value is highly dependent on the damping ratio.

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \qquad \textit{Equation } 6\ [1]$$

The damping ratio can be calculated from Equation 7; however, due to ζ being sufficiently small, a simplified version of this equation is used in this experiment shown in Equation 8 [1]. In Equation 7 and 8, x_0 and x_n indicates the displacement at the first peak and at the nth peak. The period of the response, T_d, can be calculated by finding the average time between the peaks in

Equation 9 where n_{ab} is the number of oscillations. The damped frequency can be calculated form the period by utilizing Equation 10.

$$\frac{\zeta}{\sqrt{1-\zeta}} \approx \frac{1}{2\pi n} \ln\left(\frac{x_0}{x_n}\right) \qquad \textit{Equation } 7 \text{ [1]}$$

$$\frac{\zeta}{\sqrt{1-\zeta}} \approx \zeta \approx \frac{1}{2\pi n} \ln\left(\frac{x_0}{x_n}\right) \quad for\ \zeta \ll 1 \qquad \textit{Equation } 8 \text{ [1]}$$

$$T_d = \frac{T_b - T_a}{n_{ab}} \qquad \textit{Equation } 9$$

$$\omega_d = \frac{2\pi}{T_d} \qquad \textit{Equation } 10$$

Equipment (Caroline)

The following equipment are required to perform the Secord Order System Experiment.

Table 1: Equipment List

Equipment [1]
Protective Eyewear - Used as safety for eye while doing lab activity.
Hearing Protection - Used to prevent hearing damage from the Cam Follower Experiment performed by another group.
Model 210a Rectilinear Dynamic System. (See Figure 4)
High-stiffness, medium-stiffness and low-stiffness springs. (See Figure 5)
Four 500-gram brass masses. (See Figure 6)
Allen wrenches.

Computer data acquisition board (EDyn32 Software) for recording and plotting generation of the Rectilinear Dynamic System. (See Figure 7)
Flash drive for saving the data files.

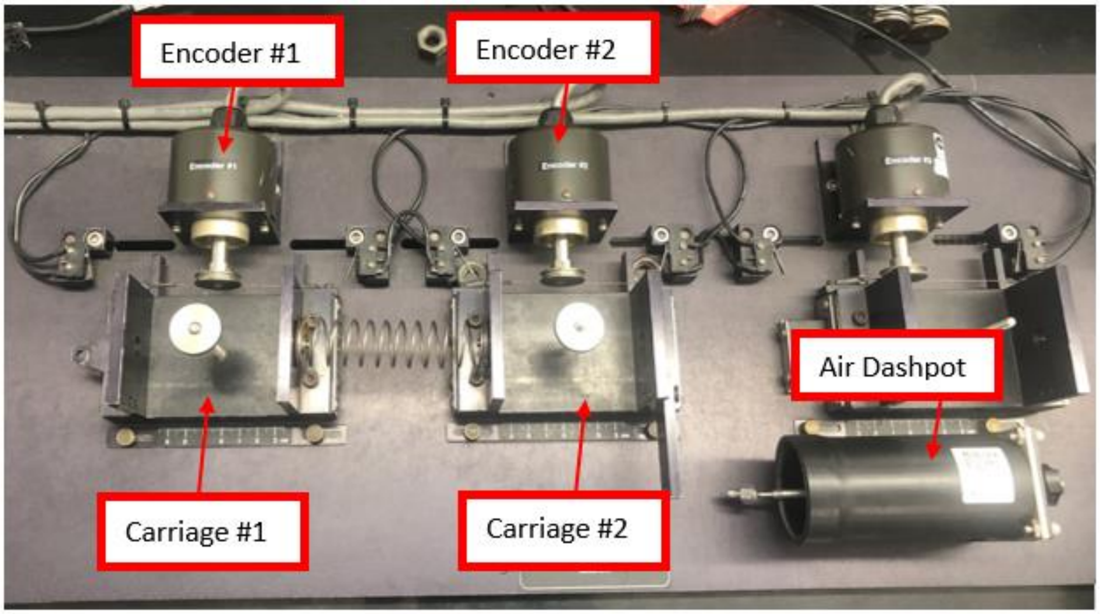

Figure 4:Model 210a Rectilinear Dynamic System

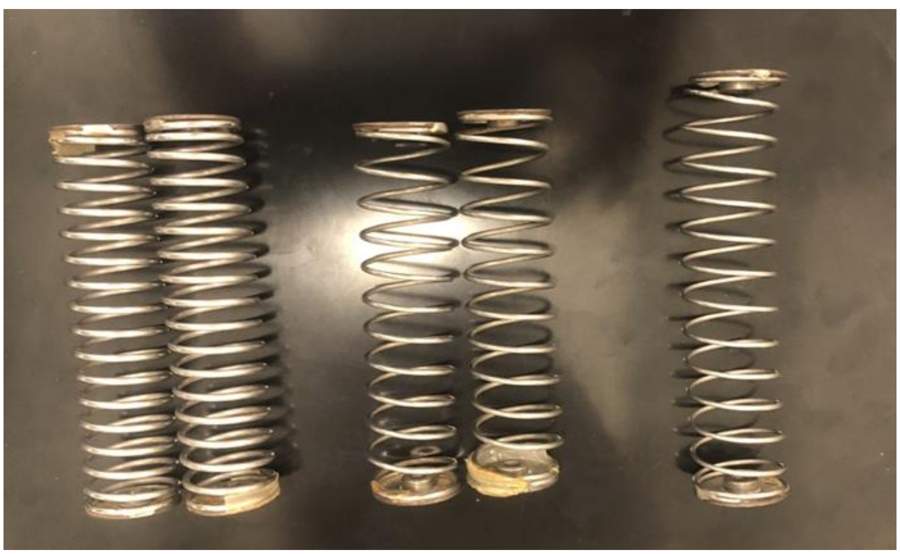

Figure 5: [From Left to Right] High-stiffness, medium-stiffness and low-stiffness springs

Figure 6: Four 500-gram brass masses

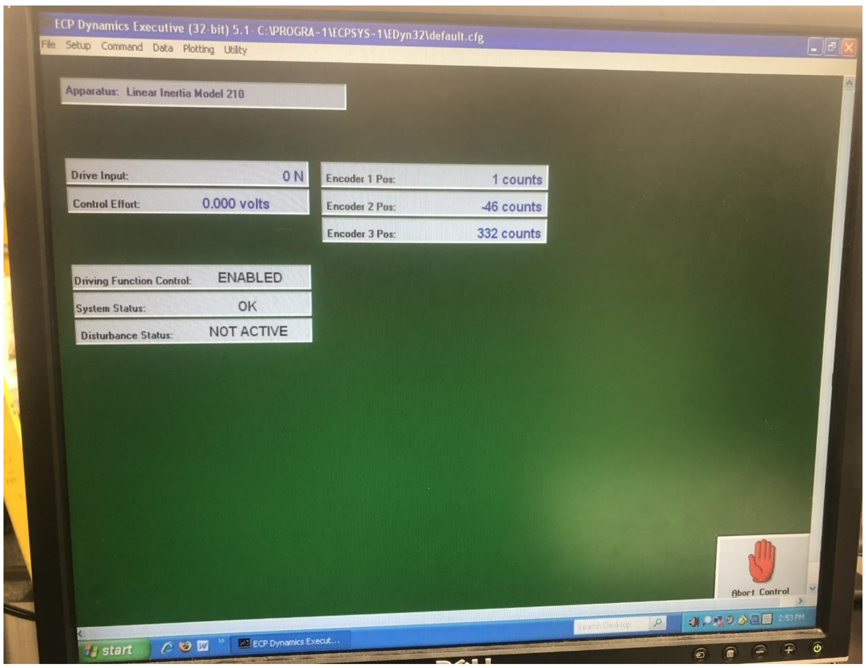

Figure 7: Computer data acquisition board (EDyn32 Software)

Procedure [1] (Caroline)

1. Launched the EDyn32 software. Entered the Driving Function box via the Setup menu and selected Force (Torque). Then selected Setup Driving Function and clicked OK, selected Enable Driving Function and clicked OK.
2. In the Setup menu, went to Input Shape, and selected Step Input. Entered a Step size of 0 (zero), duration of 3000 milliseconds and 1 repetition. Attempted to exit to the background

screen by consecutively clicking OK but received an error stating that the dwell input had to be 0 to 0.

3. To debug, the program was closed and relaunched and Steps 1 and 2 were repeated without any errors.
4. Went to Setup Data Acquisition in the Data menu and selected Encoder #1 and Encoder #2 as data to acquire, and specified data sampling every two servo cycles and clicked OK.

CASE 1

5. Securely clamped carriage #2 in place using the stop bumpers as shown in Figure 8. Then places a ¼ inch threaded nut between each bumper and the cart without engaging the limit switches on the bumpers. Made sure that the centerline mark of carriage #1 coincided with the 0 of the scale provided along carriage #1. Moved the stop bumpers for carriage #1 to the extreme outer positions. Checked that the medium-stiffness spring was between carriage #1 and carriage #2.

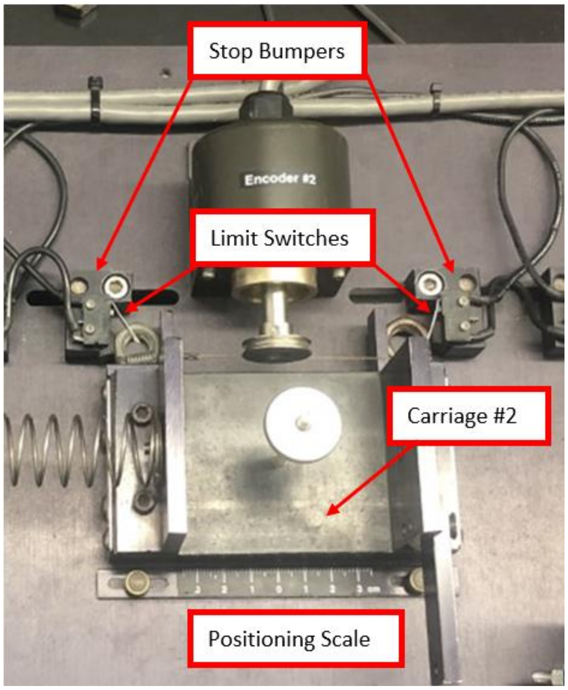

Figure 8: Detailed View of Carriage Locked in Place by Stop Bumpers

6. Secured four 500-gram masses on carriage #1. (See Figure 9)
7. The centerline mark of carriage #1 no longer coincided with the 0 of the scale provided along carriage #1 so the carriage was re-centered.

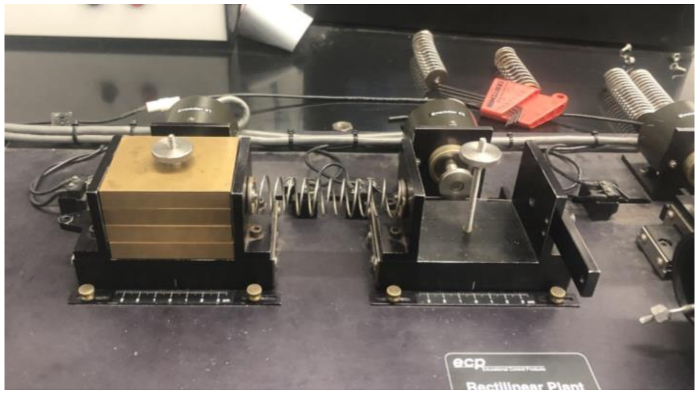

Figure 9:Complete Rectilinear Dynamic System Setup for Case 1

8. Selected Execute from the Command menu. Manually displaced carriage #1 approximately 2.5 cm carefully to not engage the limit switch. With the carriage held at this position, Run was selected from the Execute box and then the carriage was released approximately 1 second later. Clicked OK after the data was uploaded.
9. Set-up Plot was selected from the Plotting menu and Encoder #1 position was added to the left axis. Plot data was then selected from the Plotting menu. Start and ending peaks were selected and recorded in the Raw Data Sheet in Appendix 2 for analysis.
10. Export raw data was selected in the Data menu and the data was saved in a .txt format. The file was then opened in Wordpad to ensure it was correctly saved.
11. Data had Encoder #3 and Drive Input data. Went to Set-up Data Acquisition and deselected Encoder #3 and Drive Input. Then repeated Steps 8 thru 10.

CASE 2

12. Removed the extra mass from carriage #1. (See Figure 10)
13. Zeroed the carriage encoder position in the Utility menu.

14. Repeated steps 8 thru 10 for the new case.

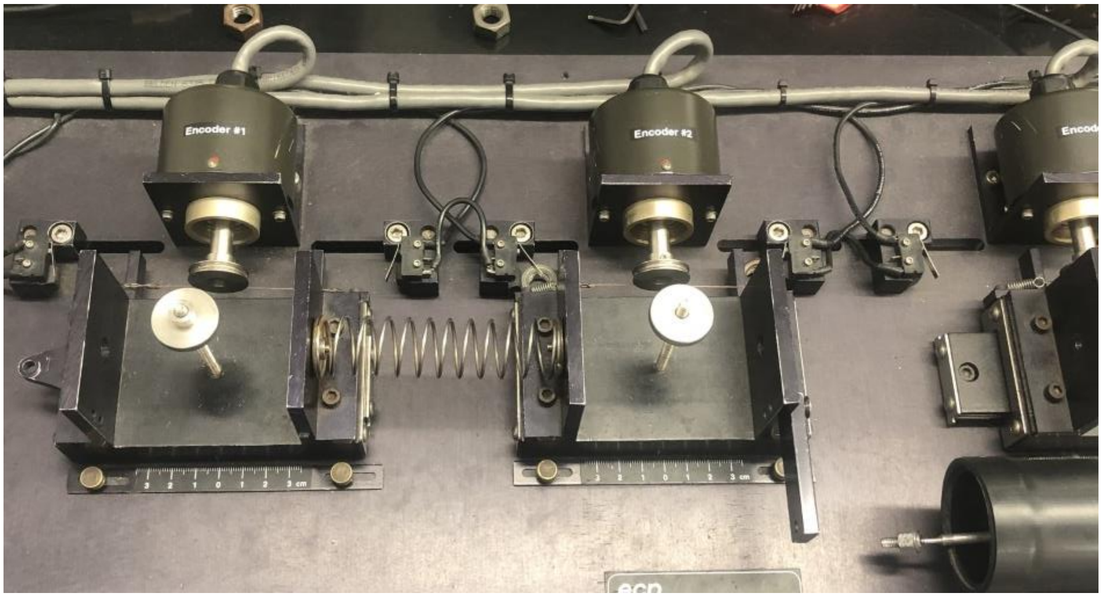

Figure 10: Complete Rectilinear Dynamic System Setup for Case 2

CASE 3

15. Unclamped carriage #2 and clamped carriage #1 using the same procedure as Step 5.
16. Secured four 500-gram masses on carriage #2. (See Figure 11)
17. In the Utility menu, selected Zero position to cero the initial encoder readings.
18. Selected Execute from the Command menu. Manually displaced carriage #2 approximately 2.5 cm carefully to not engage the limit switch. With the carriage held at this position, Run was selected from the Execute box and then the carriage was released approximately 1 second later. Clicked OK after the data was uploaded.
19. Set-up Plot was selected from the Plotting menu and Encoder #1 position was removed and Encoder #2 position was added to the left axis. Plot data was then selected from the Plotting menu. Start and ending peaks were selected and recorded in the Raw Data Sheet in Appendix 2 for analysis.

20. Step 10 was then repeated for this case.

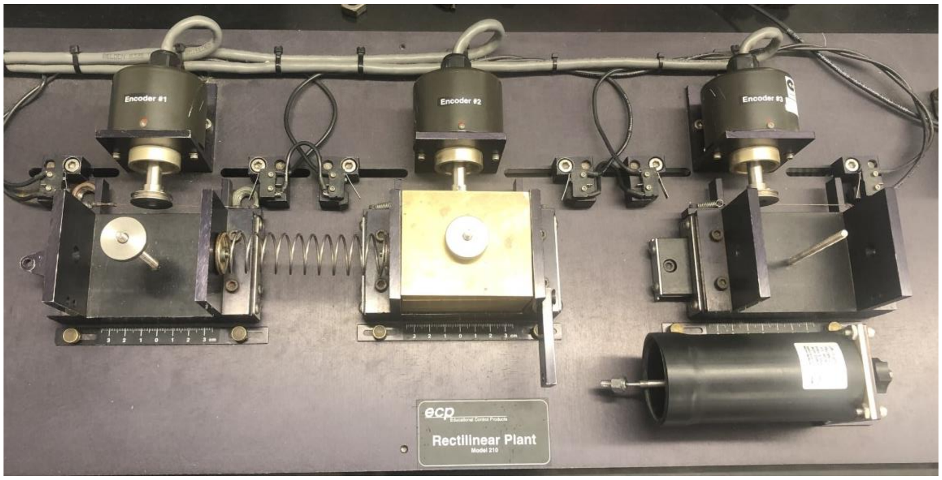

Figure 11: Complete Rectilinear Dynamic System Setup for Case 3

CASE 4

21. The extra mass was removed from carriage #2. (See Figure 12)

22. Steps 17 thru 20 were repeated for this case.

Figure 12: Complete Rectilinear Dynamic System Setup for Case 4

CASE 5

23. The dashpot was then connected to carriage #2.
24. The damping adjustment knob was set to its fully closed position without over-tightening.
25. The damping adjustment knob was then opened by making 2 complete turns from its fully closed position. (See Figure 13)

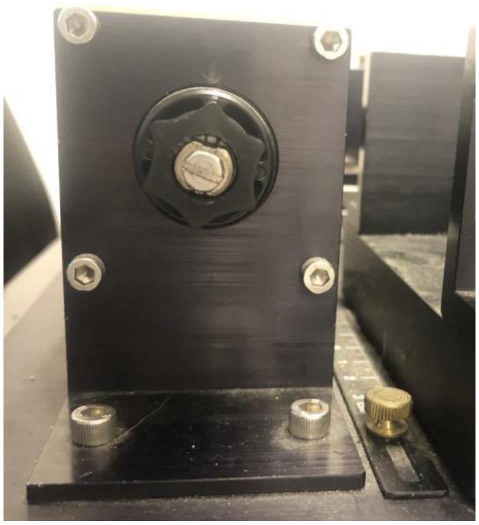

Figure 13: Damping Adjustment Knob

26. Secured four 500-gram masses on carriage #2. (See Figure 14)
27. Steps 17 thru 20 were repeated for this case.

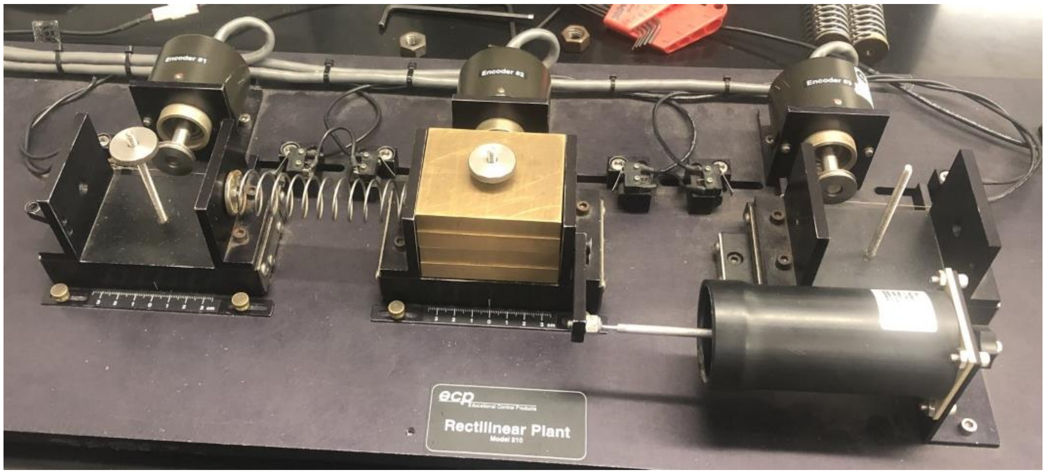

Figure 14: Complete Rectilinear Dynamic System Setup for Case 5

CASE 6

28. Disconnect the dashpot and removed the extra masses from carriage #2.

29. Replaced the medium stiffness spring with a high-stiffness spring. (See Figure 15)

30. Steps 17 thru 20 were repeated for this case.

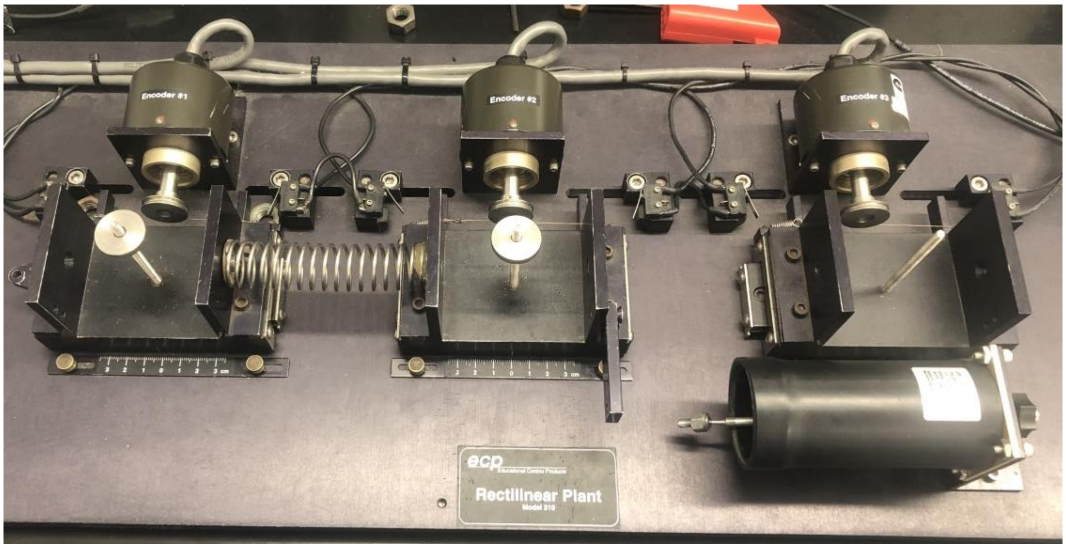

Figure 15: Complete Rectilinear Dynamic System Setup for Case 6

CASE 7

31. Replaced the high-stiffness spring with the low-stiffness spring. (See Figure 16)

32. Steps 17 thru 20 were repeated for this case.

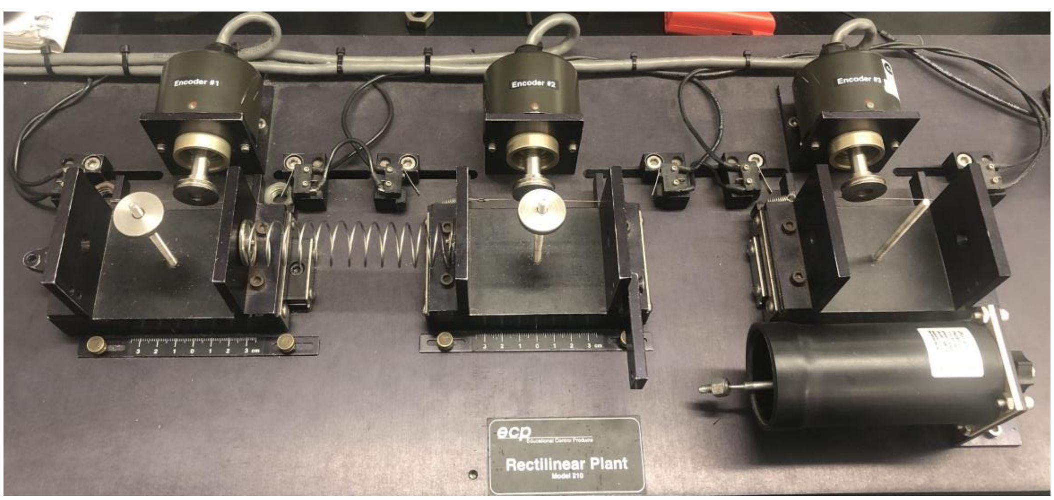

Figure 16:Complete Rectilinear Dynamic System Setup for Case 7

Results, Analysis, and Discussion (Shane)

After completing experiment for each of the 7 cases, the data was analyzed. Figures 17-23 show how the position of the unanchored carriage varied with respect to time. By viewing Figures 17-20, one can observe that the system was dampened more quickly when no added weight was used. By viewing Figures 20, 22 and 23, one can observe that the system dampened more quickly when a lighter spring was used. Case 5, shown in Figure 21, utilized a dashpot and was dampened the quickest of all.

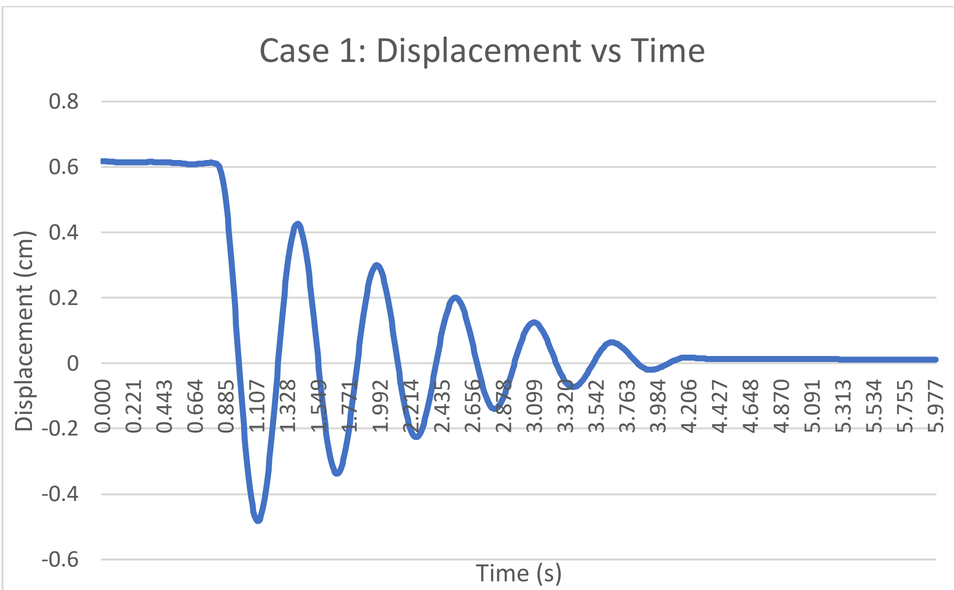

Figure 17: 2 kg Added Weight to Carriage 1

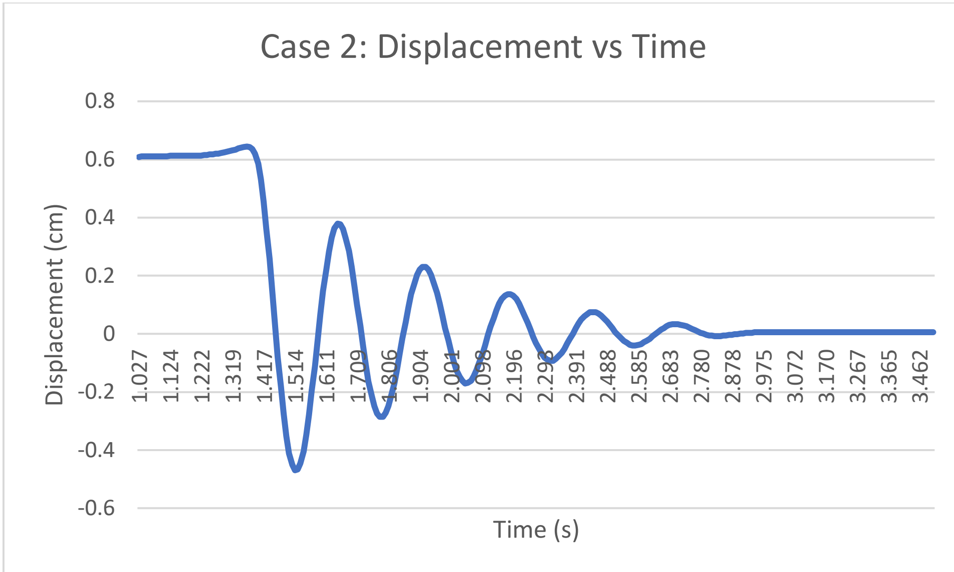

Figure 18: No Added Weight to Carriage 1

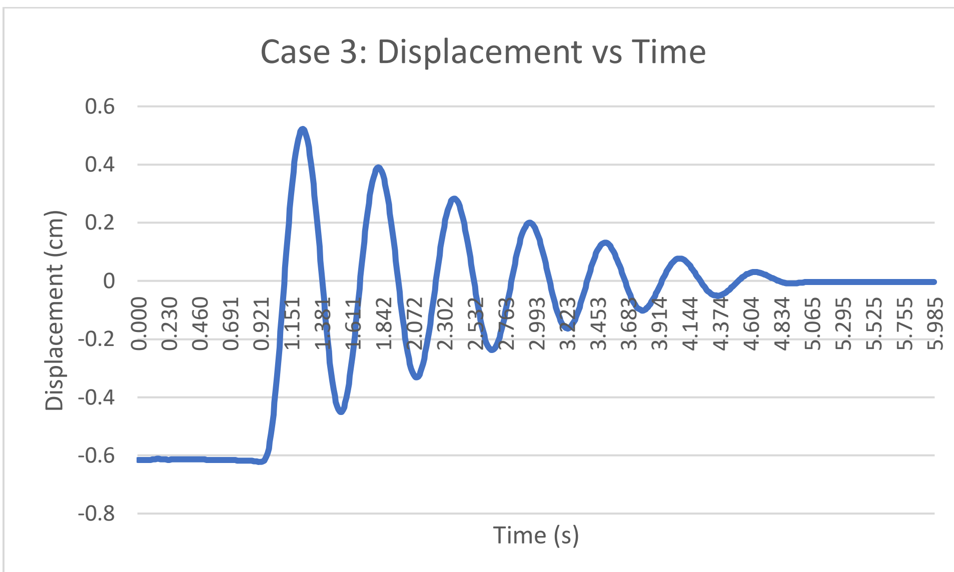

Figure 19: 2 kg Added Weight to Carriage 2

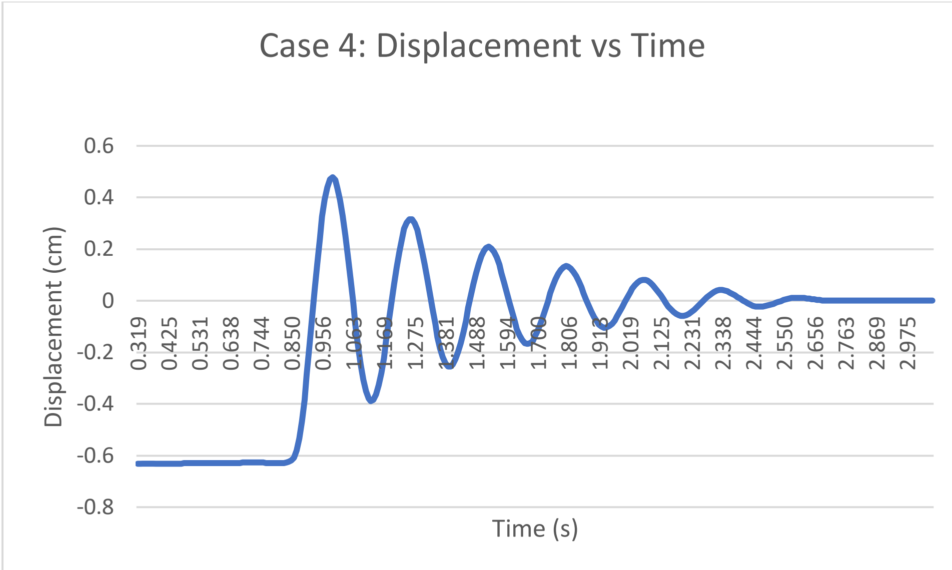

Figure 20: No Added Weight to Carriage 2

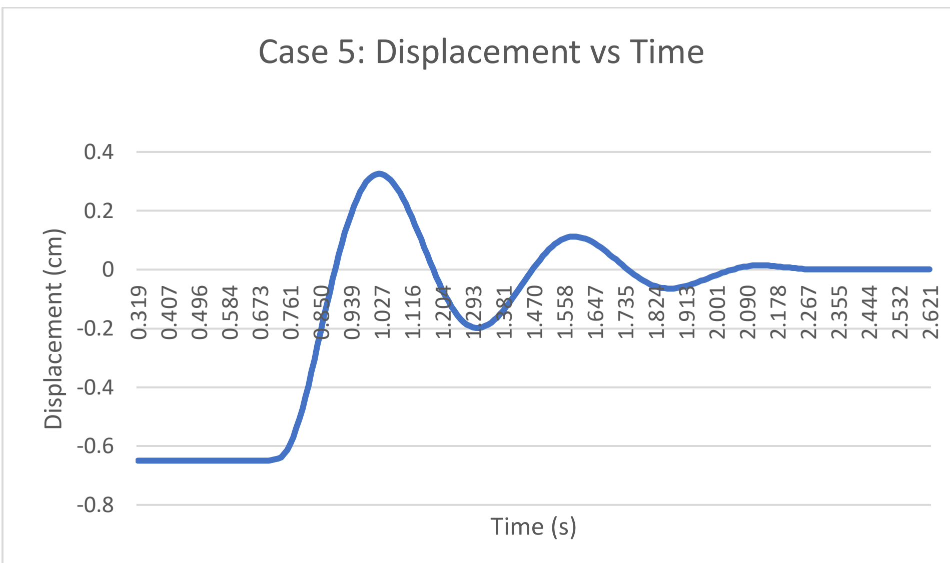

Figure 21: Dashpot attached to Carriage 2

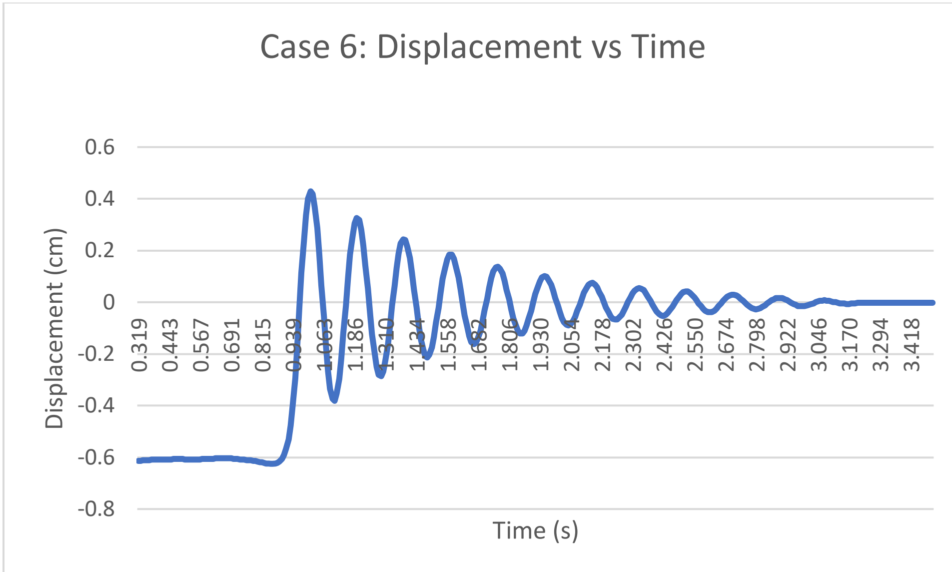

Figure 22: High Stiffness Spring Used

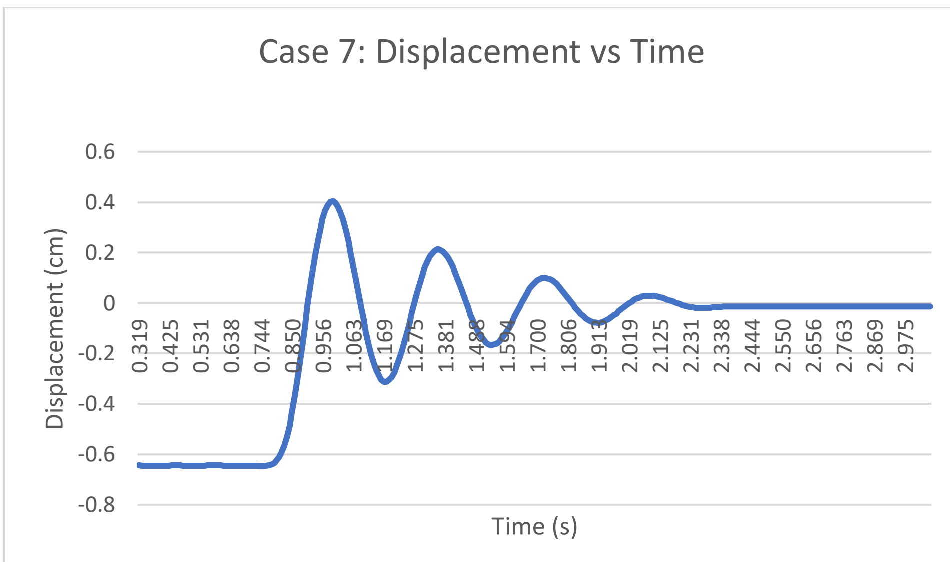

Figure 23: Low Stiffness Spring Used

Table 2 contains the damped frequency, the damping ratio, and the natural frequency of the system in each of the 7 cases. The damped frequency was calculated using equation 10 and the period of oscillation in the Position vs Time plots (Figures 17-23). The damping ratio was calculated using equation 8. The higher the damping ratio, the faster the system returned to equilibrium. The highest damping ratios occurred when a dashpot was used, when a low stiffness spring was used, and when no mass was added. In each case, the damping ratio was well below

1, meaning that the system was underdamped. The natural frequency of each case was calculated using equation 6. The cases with added mass had a lower natural frequency than the cases without added mass. Also, as shown in Table 2, the cases with higher spring stiffness had higher natural frequencies.

Table 2: Natural Frequency of Each Case

	ω_d (Hz)	ζ	ω_n (Hz)
Case 1	11.140	0.0657	11.164
Case 2	23.665	0.0807	23.743
Case 3	11.004	0.0486	11.017
Case 4	23.710	0.0660	23.762
Case 5	10.908	0.1693	11.068
Case 6	33.244	0.0453	33.278
Case 7	17.309	0.1020	17.400

The mass of carriage 1 was calculated using the known conditions in cases 1 and 2. It was known that the carriage in case 1 weighed 2 kg more than the carriage in case 2, because of the 2kg of added mass. It was also known that the same spring constant was used in both cases. Based on this, equation 3 was solved two times and the mass of the carriage was calculated. The same approach was used on cases 3 and 4 in order to calculate the mass of carriage 2. These values are depicted in Table 3.

Table 3: Mass of Carriage

	Mass (kg)
Carriage 1	0.567
Carriage 2	0.548

After finding the mass of each carriage, the 3 spring constants were calculated using equation 3. Since the medium stiffness spring had been used in cases 1 and 2, as well as in cases 3 and 4, its spring constant was calculated twice. The average value is listed in Table 4 because this was the value that was used for the remainder of the calculations. The spring constant for the high stiffness and low stiffness springs are also listed in Table 4.

Table 4: Spring Constant

	Spring Constant (N/m)
K_H	606.906
K_M	314.659
K_L	165.911

Table 5 contains the damping coefficients for carriage 1, carriage 2, and carriage 2 with a dashpot attached. These values were calculated using equation 4. The higher the damping coefficient, the faster the system returned to equilibrium. When a dashpot was attached, carriage 2 had a damping coefficient of around 477% higher than when it did not have a dashpot attached.

Table 5: Damping Coefficient

	Damping Coefficient (N*s/m)
C_{C1}	2.157
C_{C2}	1.734
C_D	10.000

Conclusions (Shane)

The natural frequency of the system depends on the spring constant and the mass of the carriage. According to equation 3, natural frequency should increase as the spring stiffness increases and should decrease as mass increases. The data recorded in Table 2 agrees with this. The smaller the mass of the carriage, and the lower the stiffness of the spring, the higher the damping ratio of the system will be. Equation 4 agrees with this statement which has also been based off of the data recorded in Table 5. The cases with the highest damping ratio were found to return to steady state the fastest, which also matches theory. Lastly, as expected, when a dashpot was attached to a carriage, an increase in its damping coefficient as found; in this experiment, a 477% increase in the damping coefficient was calculated.

These findings can be beneficial in real world applications. Trampolines can be optimized by using high stiffness springs and a heavy jumper. This will create a low damping ratio and should allow for many high amplitude jumps. These findings can also be useful to improve vehicle suspension systems and door dampers as these are situations in which one does not want a lot of oscillation [7].

One recommendation for improving this lab would be to study how different masses effect the natural frequency of the system. Natural frequency could be calculated based on the experimental results for various carriage masses and a line of best fit could be plotted. This experimental data could then be compared to the theoretical relationship between mass and natural frequency using equation 3. Perhaps this could be done in place of cases 1 and 2 which were essentially the same as cases 3 and 4, except for the fact that carriage 1 was free to move instead of carriage 2.

References

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[7] “Oscillation - Real-Life Applications.” Science Clarified, www.scienceclarified.com/everyday/Real-Life-Physics-Vol-2/Oscillation-Real-life-applications.html.

Appendix

1. **Sample Calculations (Shane)**

1) Counts → cm

$$\frac{\text{count}}{2266} = \text{cm} \rightarrow \frac{1394}{2266} = 0.615\ \text{cm}$$

2) a) $W_{d_1} = \frac{2\pi}{T_{d_1}}$ $T_{d_1} = \frac{T_b - T_a}{n_{ab}}$ n_{ab} = # of oscillations

$$T_{d_1} = \frac{2.816 - 1.124}{3} = 0.564$$

$$W_{d_1} = \frac{2\pi}{.564} = \underline{11.140\ \text{HZ}}$$

b) $\zeta = \frac{1}{2\pi n} \ln\left(\frac{x_0}{x_n}\right)$

$$\zeta = \frac{1}{2\pi(3)} \ln\left(\frac{-.482}{-.139}\right) = \underline{0.0657}$$

c) $W_n = \frac{W_d}{\sqrt{1-\zeta^2}} = \frac{11.14}{\sqrt{1-.0657^2}} = \underline{11.165\ \text{HZ}}$

3) $m_1 = m_{c_1} + m_{added}$

$K_1 = K_m$

$m_2 = m_{c_1}$

$K_2 = K_m$

$K_1 = K_2 = K_m$

$m_{added} = 2000g$

$W_{n_1} = \sqrt{\frac{K_1}{m_1}}$ $W_{n_1} = 11.165\ \text{Hz}$

$W_{n_2} = \sqrt{\frac{K_2}{m_2}}$ $W_{n_2} = 23.743\ \text{Hz}$

$$23.743^2 = \frac{K_2}{m_2} = \frac{K_m}{m_{c_1}}$$

$$563.73\, m_{c_1} = K_m$$

$$11.165^2 = \frac{K_m}{m_{c_1} + m_{added}}$$

$$124.657(m_{c_1} + m_{added}) = K_m$$

$$124.657 m_{c_1} + 124.657 m_{added} = 563.73 m_{c_1}$$

$$124.657 m_{added} = 439.073 m_{c_1}$$

$$m_2 = m_{c_1} = \underline{567.420\ g}$$

$$\underline{m_1 = 2567.420\ g}$$

$$23.742^2 (m_{c_1}) = K_2$$

$$K_1 = K_2 = 320097.349\ \frac{g}{s^2}$$

$$K_1 = K_2 = 320.097\ \frac{kg}{s^2} = 320.097\ N/m$$

3 (cont) $m_3 = m_{c2} + m_{added}$ $\quad$ $m_4 = m_{c2}$

$w_{n3} = 11.017\ Hz$ $\quad$ $w_{n4} = 23.762\ Hz$

$w_{n3}^2 (m_3) = K_m$ $\quad$ $w_{n4}^2 (m_4) = K_m$

$$11.017^2 (m_{c2} + 2) = 23.762^2\ m_{c2}$$

$$121.374\, m_{c2} + 242.75 = 564.633\, m_{c2}$$

$$\underline{m_{c2} = .548\ Kg} = m_4 = m_6 = m_7$$

$$m_3 = 2.772\ Kg = m_5$$

$$w_{n4}^2 (m_4) = K_m$$

$$K_m = 309.22\ N/m$$

$$K_{mavg} = \frac{320.097 + 309.22}{2} \qquad \underline{K_{mavg} = 314.659\ N/m}$$

$$\zeta = \frac{c}{2\sqrt{mK}}$$

no added mass

$$\zeta_2 = \frac{c_{c1}}{2\sqrt{m_2 K_m}} \rightarrow \underline{c_{c1} = 2.157\ \frac{N \cdot s}{m}}$$

no added mass

$$\zeta_4 = \frac{c_{c2}}{2\sqrt{m_4 K_m}} \rightarrow \underline{c_{c2} = 1.734\ \frac{N \cdot s}{m}}$$

$$\zeta_5 = \frac{c_D}{2\sqrt{m_5 K_m}} \rightarrow c_D = .1693\left(2\sqrt{2.772(314.659)}\right) \rightarrow \underline{c_D = 10.000\ \frac{N \cdot s}{m}}$$

$$w_n = \sqrt{\frac{K}{m}}$$

$$w_{n6} = \sqrt{\frac{K_H}{m_6}} \rightarrow K_H = 33.279^2 (.548) \rightarrow \underline{K_H = 606.906\ \frac{N}{M}}$$

$$w_{n7} = \sqrt{\frac{K_L}{m_7}} \rightarrow K_L = 17.3999^2 (.548) \rightarrow \underline{K_L = 165.911\ \frac{N}{m}}$$

2. Raw Data Sheets (Mallory)

Data Sheet: Lab 6 $w_d = w_n(0.99745)$ $\quad 2.9 - 1.05 = \frac{1.85}{3} = .617$

$\zeta = 0.07$

Case 1:

added mass on 1

$x_0 = -1150$ counts =

$x_n = -300$ counts =

$t_0 = 1.05$ sec

$t_n = 2.9$ sec

# of cycles: $n = 3$

$w_d = 10.2$ Hz

$w_{d_1} = \frac{2\pi}{T_{d_1}} = \frac{2\pi}{0.616} = \cancel{9.92}\ 10.2$ Hz

Case 2:

no mass

$x_0 = -1050$

$x_n = -200$

$t_0 = 1.5$

$t_n = 2.3$

$n = 3$

$W_{d_2} = 23.56$ Hz

Case 3:

added mass on 2

$x_0 = 1200$

$x_n = 300$

$t_0 = 1.3$

$t_n = 3.5$

$n = 4$

$w_{d_3} = 11.42$ Hz

Case 4:

no mass

$x_0 = 1100$

$x_n = 200$

$t_0 = 1$

$t_n = 2.1$

$n = 4$

$w_{d_4} = 22.85$ Hz

mass on 2 and damped

Case 5:

$x_0 = 750$

$x_n = 250$

$t_0 = 1$

$t_n = 1.7$

$\eta = 1$

$\omega_{d5} = \frac{2\pi}{.7} = 8.98\,Hz$

heavy spring

Case 6:

$x_0 = 1000$

$x_n = 100$

$t_0 = 1sec$

$t_n = 2.4$

$\eta = 7$

$\omega_{d6} = 31.42\,HZ$

light spring

Case 7:

$x_0 = 900$

$x_n = 250$

$t_0 = 1$

$t_n = 1.8$

$\eta = 2$

$\omega_{d7} = 15.71\,Hz$

3. Procedure (Caroline)

Procedure

1) launched ECDyn32 Software, steps followed for step one in procedure in lab manual.

2) attempted step two in procedure and received an error. close and relaunched program and was able to carry out step two. (error was regarding the dwell input had to be 0 to 0)

3.) followed step 3 in the lab manual.

case 1

4 placed nuts in correct position, followed step 4 in Lab manual (see picture) for setup configuration

5) secured 500g masses on carriage #1 (see picture)

6.) followed step 6 in lab manual

7. recentered carriage 1

8. Step 7 and 8 from the lab manual were conducted. see raw data sheet for selected estimates

9. step 9 in the lab manual was conducted and saved.

10. data had encoder # 3 & drive input. Went to data, setup data acquisition and deselected them, Then repeated steps 8 and 9 with new data. see raw data sheet.

Procedure Continued

Case 2

11.) removed extra masses from carriage 1 (see picture) followed step 12 in lab manual.

12) repeated steps 7-10 in the lab manual (see raw data sheet)

Case 3

13.) step 14 and 15 from the lab manual were followed. (see picture)

14.) in the utility menu the zero position was selected to zero the initial encoder readings

15.) Step 17, 18, 19 and 20. (graph is upside down because carriage 2 was pulled to the right instead of to the left. see raw data sheet.

Case 4

16.) removed extra masses from carriage 2 (see picture)

17.) Steps 16-20 in the lab manual were repeated. (see raw data sheet)

Case 5

18.) The dashpot was connected to carriage 2

19) steps 24-26 from the lab manual were conducted. (see pictures)

20) Steps 16-20 in the lab manual were repeated (see raw data sheet)

Procedure Continued

Case 6

21.) Disconnected the dashpot and removed the extra mass from carriage 2.

22) replaced the medium stiffness spring with a high-stiffness spring (see picture)

23) enable driving function (limit switch was engaged.)

24.) Steps 16-20 in the lab manual were repeated (see raw data sheet.)

Case 7

25) Replaced the high-stiffness spring w/ the low-stiffness spring (see picture)

26) Steps 16-20 in the lab manual were repeated (see raw data sheet)